

# For Want of a Nail:

## An Introduction to Functions, Ordered Pairs, and Coordinate Planes



Grades 5–10

Using AIT Products

- *Music to My Ears*, program 4, “Zoom In, Zoom Out”
- *Economics at Work*, Module 2: Exchanging, “Build You a ‘Vette?’”
- *Oxygen: What a Gas!*, program 1, “The Breath of Life”
- *It Figures*, program 19, “Finding Equivalent Fractions”
- *Mathemedia*, program 12, “Coordinates”

### Overview

This lesson is designed to help students simplify, and therefore fully understand, the nature of algebraic functions and relationships. It can serve as introductory information for students taking pre-Algebra or as a refresher for students in Algebra I who haven’t yet mastered a deep understanding of the concept of function.

According to the National Council of Teachers of Mathematics (NCTM), function is “. . . probably the single most important idea in mathematics. Functions are pervasive in mathematics, and the concept links together many of its branches. Integrating the study of functions throughout the curriculum is a powerful means of connecting mathematics.”<sup>1</sup> This lesson uses clips from AIT videos to help students connect functions throughout the curriculum. The videos encour-

age them to identify relationships as examples of functions in music, economics, and health. Students will then generalize that understanding to functions, ordered pairs, and linear equations in mathematics. They will build deep understanding of linear functions and equations by graphing solutions of linear equations on coordinate planes and analyzing the data represented there to solve real-world problems.

**Prerequisite skills:** Students should be able to perform basic calculations that involve adding, subtracting, and multiplying whole numbers, fractions, and decimals.

<sup>1</sup> Froelich, Gary W., Kevin G. Bartkovich, and Paul A. Foerster. *Connecting Mathematics (Curriculum and Evaluation Standards for School Mathematics Addenda Series, Grades 9-12)*. Reston, Virginia: The National Council of Teachers of Mathematics, 1991.

## Objectives

- Define the term *function* and explore functions as cause/effect relationships in a variety of settings.
- Identify and analyze linear functions from data found in ordered pairs and linear equations.
- Create and solve linear equations using symbolic algebra to represent situations involving linear relationships.
- Graph and analyze simple linear equation solution sets on a coordinate plane.

## Vocabulary

braces	graphing
cause	inputs
coordinate plane	linear equations
effect	ordered pairs
expressions	outputs
franchise	plotting
franchise fee	relations
functions	solution set

## Preparation

### Materials Needed

- **For Want of a Nail** poster
- AIT video *Music to My Ears*, “Zoom In, Zoom Out”—CUE the tape to approximate time code 01:31 (about 2 minutes in total length).
- AIT video *Economics at Work*, “Build You a ‘Vette?”—CUE the tape to approximate time code 02:08 (a little over 1 minute in total length).
- AIT video *Oxygen: What a Gas!*, “The Breath of Life”—CUE the tape to approximate time code 02:24 (about 3.5 minutes in total length).
- AIT video *It Figures*, “Finding Equivalent Fractions”—CUE the tape to approximate time code 08:18 (about 3 minutes in total length).
- **The Input/Output Machine** handout, one for each student
- **André’s Trip Home** worksheet, one for each student
- AIT video *Mathemedia*, “Coordinates”—CUE the tape to approximate time code 00:36 (less than 3 minutes for Day Two; see below for second clip from this video).
- AIT video *Mathemedia*, “Coordinates”—CUE the tape to approximate time code 12:50 (less than 6 minutes for Day Three).
- OPTIONAL: calculators (This might include the calculator feature on a computer or a calculator designed for overhead projection as well as the number of calculators you wish to provide for individual student use.)
- Graph paper and straight edges

### Planning Notes

- Prepare the **For Want of a Nail** poster for overhead transparency or computer projection.
- OPTIONAL: Prepare the **André’s Trip Home** worksheet for overhead transparency or computer projection.
- OPTIONAL: The following images are embedded within the lessons of the specified days, and may be useful projected for students to see.
  - \* Day One: The input/output machine (If you wish, you may choose to draw your own function machine on the board.)

- \* Day Two: The completed grid (answer key) from the André’s Trip Home activity
- \* Day Three: Melting Profits Graph 1
- \* Day Three: Melting Profits Graph 2
- \* Day Three: Assignment answer key: **Get Your Money’s Worth** graph

### Time

This project will take about three 60-minute class periods, in addition to homework and extension activity time.

## Procedure—Day 1

### Introduce Topic: Understanding Relationships and Functions

Project the **For Want of a Nail** poster and read the poem aloud. Working as a class, identify the cause-and-effect relationships illustrated within the poem, creating the following graphic organizer to help students visualize the relationships as you fill in the information.

CAUSE	EFFECT
Lost a nail for horseshoe	Shoe falls off

Ask students if they can guess what this poem has to do with math (algebra). Look for answers that demonstrate student understanding of how relationships apply to all parts of life, especially in math. Lead the discussion so that students conclude that many fields (social studies, science, and math, for example) depend on an understanding of relationships.

### Pre-Viewing Activity

Continue the discussion of relationships in all fields of study by asking students a series of questions similar to the following suggestions (select questions that best fit the ages and experiences of your students).

- If the general in charge of this battle had understood the relationship between the horseshoe nail and the outcome of the battle, how might he have used that information?
- How do car makers use an understanding of the relationship between cost of parts and profit?
- How do your parents use an understanding of the relationship between sleep and mental abilities?
- How do economists use an understanding of the relationship between interest rates and inflation?
- How do farmers use an understanding of the relationship between soil nutrients and healthy crops?

### Video

Prepare students for watching the AIT videos by explaining that you want students to identify

$$\begin{array}{r}
 2a + 4b = 18 \\
 2a - 3b = 4 \\
 \hline
 7b = 14 \\
 b = 2
 \end{array}$$

“Do not worry about your difficulties in mathematics. I assure you that mine are greater.”

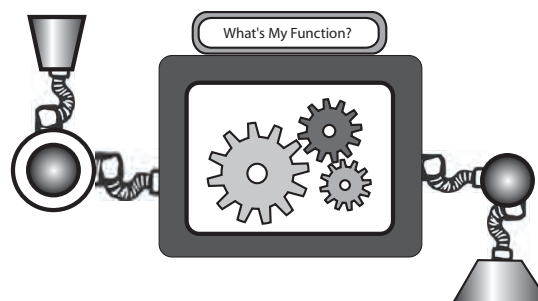
—Albert Einstein

relationships in several different settings to see how one object or event changes another. They will then see how the same principal applies in math. Create a table like the one illustrated at the bottom of this page, and then show students the four clips, stopping after each to fill in the missing part of the cause/effect relationships on the chart. Discuss how the people in the videos used their understanding of the relationships. The combined video clips should take about 10 minutes in viewing time.

- *Music to My Ears*, “Zoom In, Zoom Out” (Beginning at approximate time code 01:31, when the host begins, “So let’s join the Fairfield Four as they sing . . .” to 03:35, the end of the song.)
- *Economics at Work*, “Build You a ’Vette?” (Beginning at approximate time code 02:08, as the narrator says, “The Corvette is assembled in Bowling Green . . .” to 03:22, as the screen switches to the question.)
- *Oxygen: What a Gas!*, “The Breath of Life” (Beginning at approximate time code 02:24 with the animation showing oxygen in the blood, to 05:53, after Sir Edmund Hillary concludes, “. . . I was doing things a great deal more slowly.”)
- *It Figures*, “Finding Equivalent Fractions” (Beginning at approximate time code 08:18 with the on-screen title “Robin Hood” to the end of the animation at about 11:13.)

### Whole Group Discussion

Either project a copy of the machine graphic from The **Input/Output Machine** worksheet or draw a similar machine on the board. Explain that this machine can also show relationships similar to the cause/effect tables students have already examined. Illustrate this with the video examples students just watched, writing information in the input and output areas of the machine for each example.



- This is called a function machine. On the left side you see a place where something can be put into the machine. We’ll call that the “input.” On the right side of the machine the finished product comes out. We can call that the “output.”
- What the machine does is apply a function, or rule, to the input. No matter what we put into the machine, the machine will apply the same function or rule to that item, spitting out the finished product, or output, at the other end. In other words, the machine changes (transforms) the input into the output.

Video Clip	CAUSE	EFFECT
<i>Music to My Ears</i>	Bass singer requests that lead singer “take it up a key.”	
<i>Economics at Work</i>		The cost of cars is kept down.
<i>Oxygen: What a Gas!</i>		“I was doing things a great deal more slowly.”
<i>It Figures</i>	Robin Hood understands the relationship between equivalent fractions.	

- Think about the video we watched about the singers. Let's say the song is the input. A song is made up of notes that can be played or sung in a variety of ways, different pitches, speeds, keys, and so on. We already know what the “machine” did in this case; the group took the song “up a key.” What was the output? (The song sounded better to the singers and listeners.) You could say that the quality of the song was a function of the higher key.
- In the second video manufacturers were assembling cars (input). We already know the output was that the cost of the cars was kept down. Controlling the cost of the cars is a function of what? (Manufacturers found parts that were the best quality at the lowest price.)

Fill in the input, function, and output for the videos from *Oxygen: What a Gas!* and *It Figures*. Be sure to end each discussion with a question, such as “Moving more slowly is a function of what?” Then have students think back to the poem (“For Want of a Nail”) that began this discussion. “The loss of the kingdom is a function of . . .” (a missing horseshoe nail).

Introduce mathematical symbols for functions with the following steps. Write the phrase “ $y$  is a function of  $x$ ” on the board and be sure to stress it often to help students internalize this common algebraic expression.

- From what we've just seen, you could say that a function is a rule that changes something else. When talking about functions in math, it's easy to substitute symbols for some of this terminology. Let's begin thinking of the input as  $x$ . The output will become  $y$ . Remember, the machine changes  $x$  into  $y$ . Therefore,  $y$  is a function of  $x$ . In math, the machine uses an equation to make that change.
- Whenever we identify both the  $x$  and  $y$  of a function in math, we're going to start collecting them in “ordered pairs” inside parenthe-

ses, like this:  $(x, y)$ . In an ordered pair, the input ( $x$ ) changes into the output ( $y$ ) after the function or rule is applied to it.

- Let's say that you input a number, 24, into the Input/Output Machine, and the number 6 came out the other end. (Write “24” above the input funnel on the machine and “6” below the output funnel.) What might the machine have done to the number 24 to change it to 6? (If students jump to a solution too quickly, ask one or more of the following questions, depending on their guesses, to refocus their thinking.)
  - \* But what if the function involved subtraction? [ $x$  minus 18]
  - \* But what if the function involved fractions? [ $\frac{1}{4}$  of  $x$ ]
  - \* But what if the function involved division? [ $x$  divided by 4]
- Because we can come up with at least three possible functions for this ordered pair, it's obvious that we need to look at more input/output relationships—ordered pairs—to really know what's going on inside the machine. Here are three more ordered pairs that make up this machine's “solution set”:  $\{(12, 3), (32, 8), (60, 15)\}$ . (Point out the braces that enclose the three ordered pairs and make sure all students understand the term *solution set*.) Now can you find the function? (Divide by 4.)
- If I were to write an equation using the symbols  $x$  and  $y$  to show the function or rule this machine applies, I would write  $x / 4 = y$  (or  $x \div 4 = y$ ).

If necessary, create one or two more examples to give students additional practice before assigning the following worksheet.

## Homework: Ordered Pairs and Functions

Divide the class into small groups of three or four students and provide each student with a copy of **The Input/Output Machine** worksheet. Have the groups work together to complete part A of the worksheet. As a class, review the correct answers for part A:

1. The function of the machine could be written as the equation  $\frac{x+3}{3} = y$  (or another multiplication/fraction format, depending on your class's experience with the symbols).
2. The solution set of ordered pairs resulting from that function is  $\{(2/3, \underline{6/9}), (1/6, \underline{3/18}), (3/7, \underline{9/21})\}$ .

Assign the remainder of the worksheet for individual student homework. For younger students, you may wish to reduce the number of problems, which are ordered by increasing skill levels, or you may allow students to use calculators. See the answer key at the top of page 7 for a description of math computation skills covered in each problem.

## Procedure—Day 2

### Review/Reflection

Have students exchange homework papers and check answers as a group (see the answer key at the top of page 7).

Pay particular attention to student responses to the Bonus Question of the worksheet. Students should have had enough practice with functions, ordered pairs, and solution sets to SYNTHESIZE and APPLY their understanding, realizing that the answer is “no”—because every function associates a unique output ( $y$ ) with every input ( $x$ ) drawn from a fixed set. (Students might have explained their answers in a variety of ways; including the simple explanation that there can-

not be two different solutions to the same math problem:  $2 + 2$  is always 4.)

Write a few solution sets like those suggested below on the board and randomly call on students to tell you if the sets of ordered pairs make up a function. If they do, what is the function? If not, what keeps them from meeting the criteria? (Make sure everyone understands this concept before continuing with the new topic.)

- $\{(2, 4), (4, 3), (1, 3), (2, 1)\}$
- $\{(5, 20), (3, 12), (2, 8), (6, 24)\}$
- $\{(27, \frac{1}{3}), (12, \frac{1}{3}), (12, \frac{1}{6}), (30, \frac{1}{3})\}$

Review the terminology and concepts students have learned so far, including *relations*, *input*, *output*, *functions*, *solution sets*, and *ordered pairs*.

### Introduce New Topic: Plotting Ordered Pairs on a Coordinate Plane

In the activities described on Day One, students learned to recognize simple functions, where a given number related to another number because of a mathematical calculation. In this activity they will be taking that information further to understanding linear equations. Write the following two equations from the worksheet side by side on the board, and then write the second equation below each.

$$x - 8 = y \qquad 10x - 1 = y$$

$$f(x) = x - 8 \qquad f(x) = 10x - 1$$

Ask students to guess how the new equations relate to the original functions. If no one guesses correctly, explain that the two written forms are the same—the term  $f(x)$  means “the function of  $x$ ” and takes the place of  $y$  in the equation. Remind students again of the phrase they explored on Day One:  $y$  is a function of  $x$ . Since  $y$  is a function of  $x$ , the symbol  $f(x)$  takes the place of  $y$  in the function. (This also might be a good

<b>The Input/Output Machine worksheet—Answer Key</b>	
<b>MATH SKILLS REQUIRED</b>	<b>ANSWERS</b>
1. (Skill: basic addition facts)	10, 28, [ $x + 7 = y$ ]
2. (Skill: basic subtraction facts)	11, 4, [ $x - 8 = y$ ]
3. (Skill: basic multiplication facts)	33, 15, [ $3x = y$ ]
4. (Skill: multiplying decimals)	2.00, 25.00, [ $.25x = y$ ]
5. (Skill: multiplying fractions)	$\frac{2}{9}, \frac{4}{15}, [x \cdot \frac{2}{3} = y]$
6. (Skill: adding fractions with unlike denominators)	$\frac{5}{16}, [x + \frac{1}{4} = y]$
7. (Skill: 2-step multiplication and addition)	11, 16, [ $2x + 1 = y$ ]
8. (Skill: 2-step multiplication and subtraction)	69, 99, [ $10x - 1 = y$ ]

time to assess students' understanding of the symmetric property of equality—if  $a = b$ , then  $b = a$ .) The first equation is read as “the function of  $x$  is  $x$  minus 8,” and the second as “the function of  $x$  is 10 times  $x$  minus 1.” The same solution sets of ordered pairs which students worked with in their worksheets match these algebraic expressions.

### Pre-Viewing Activity

Draw a simple 10-by-10 unit grid on the board, labeling the  $x$ -axis with letters A–J and the  $y$ -axis with the numbers 0–10 (or project a sheet of grid paper if you prefer, setting off the required area). Before beginning the next discussion, secretly write down the coordinates of three adjacent points on the grid; for example:  $\{(C, 4), (C, 5), (C, 6)\}$ .

Ask students if they've ever played the game called *Battleship* (go over the procedure for identifying points on the grid, if necessary), and explain that you have hidden a “cruiser” somewhere in the ocean represented by this grid. A cruiser is three segments long, and students must hit all three points to sink it. Let students take turns calling out coordinates to bomb your ship. Place circles for non-hits and Xs for hits at the points for each coordinate suggested. Continue playing until the class has “sunk your

battleship.” Watch to make sure students name the horizontal coordinate (letter) first.

### Video

Prepare students for watching the video clip from *Mathemedia* by explaining that graphs and grids are especially helpful for understanding functions, ordered pairs, and solution sets. Have students watch the video clip from “Coordinates,” beginning immediately after the credits, from the black screen at approximate time code 00:36. PAUSE the video at the first question (approximate time code 02:59): “What will the coordinates of the black checker be after it jumps the other pieces in one move?” Ask for volunteers to answer the question (make sure students know how to jump in checkers). Check to see that students understand that the letters stand for the  $x$ -value, just like in *Battleship*, and are therefore named first in the coordinate, or ordered pair.)

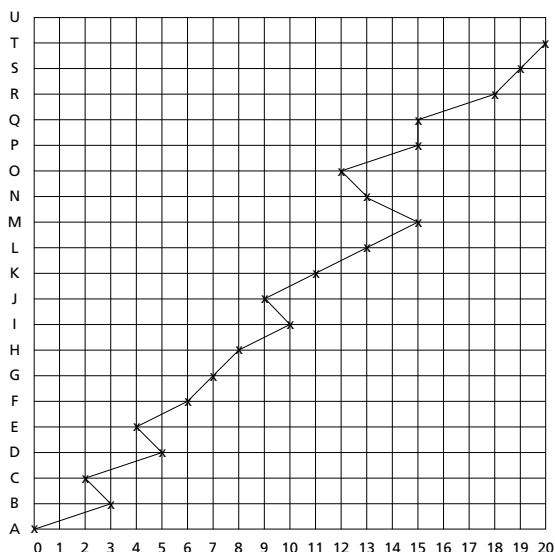
Continue to PLAY the video to allow students to check the first answer, and then PAUSE once again at approximate time code 03:18, with the second on-screen question: “Using this coordinate system, where will the black checker end up after jumping the reds?” Explain that the grid they are looking at now is called a *coordinate plane*, and they will be using coordinate planes

to graph linear functions. Once again, the  $x$ -value is named first in the ordered pair. Have volunteers answer the question, then PLAY the video to approximate time code 03:29 to check the answers, STOPPING when the scene fades to black.

### Class Work

Project a copy of the **André’s Trip Home** worksheet and read the story aloud. (NOTE: This activity was taken from the guide that accompanies the *Mathemedia* video clip.) Point out that each of the numbered pairs below the grid resembles the ordered pairs in the solution sets they’ve been working with. (Make sure students notice that in this graph the  $x$ -value in the ordered pair is a number, located along the horizontal axis. The  $y$ -value is represented by a letter on the vertical axis. Because the  $x$ -value must be named first, their ordered pairs will be the opposite of the *Battleship* and checker examples.)

Pass out copies of the worksheet to students and help them plot the first two pairs on the grid. Then give students 5–10 minutes to complete the worksheet, walking around the room to make sure everyone understands how to plot the points. The completed grids should look like this one. You may wish to project the completed grid and allow students to check their own or their neighbor’s work.



## Procedure—Day 3

### Review/Reflection

Write a simple equation like this one on the board, and ask students to suggest numbers that might be used in place of  $x$  and  $y$  to make the function true.

$$2x + 3 = y$$

As students come up with values for  $x$  and  $y$ , write them in the form of a solution set of ordered pairs on the board under the function. Try to get at least five suggestions before moving on, but don’t attempt to put the ordered pairs into any type of order. For example, possible responses for this function might include  $\{(3, 9), (1, 5), (10, 23)\}$ . Ask students to identify the following terms as they apply to the equation: *ordered pairs*, *solution set*, and *function*. Also ask students to complete the following statements using what they now know about algebraic expressions.

- You can rewrite the equation as the function \_\_\_\_\_.
- The function of  $x$  is \_\_\_\_\_.
- The input is \_\_\_\_\_ and the output is \_\_\_\_\_.

### Introduce New Topic: Graphing Linear Functions and Equations

Explain that the equation they just reviewed is something called a *linear function*. A linear function takes the form  $f(x) = mx + b$  where  $m$  and  $b$  are some fixed numbers. Ask, what is the value of “ $m$ ” in the above equation? (2) What is the value of “ $b$ ”? (3) Although the example shows an addition problem, any operation will fit equally well.

Draw a simple 15-by-15 unit grid on the board, labeling both the  $x$ -axis and  $y$ -axis with the odd numbers from 1–29 (1, 3, 5, 7, etc.). Ask for volunteers to come up and plot the points of the ordered pairs in the solution set, helping them to

locate any even numbers on the  $x$ -axis in the area between points. Ask students if they notice a pattern in the location of the ordered pairs. Use a straight edge to demonstrate how the points fall on a line. Explain that functions of this kind are called “linear” because their graphs are straight lines. Remind students of the line they created on the **André’s Trip Home** worksheet—does that graph show a linear function? (No, the line was not straight.) If necessary, plot solution sets for one or two more linear functions before continuing with the video.

## Video

Prepare students for watching the second video clip from *Mathemedia* “Coordinates” by providing a simplified explanation of the economic concept of a *franchise*. Write the term on the board and explain the basics about this type of business with the following points.

- A franchise is a very popular method for people to start a business. Let’s explore why.
- Say you want to open a fast-food burger restaurant. You could start from scratch, coming up with your own name and paying for a building, deciding on the menu, looking for suppliers of ingredients, paper goods, equipment, furniture, and so on. Then you’d have to pay for signs and advertising, etc. What are some other things you have to include in a restaurant of this kind?
- Rather than doing all the decision making yourself, you could buy a franchise from an established company like McDonald’s® or Burger King®. For the up-front price, called a *franchise fee*, those companies will provide you everything you need to begin your restaurant—sometimes even providing the building. All you have to do is pay them the wholesale cost of the goods (burger patties, paper goods, and so on).
- What are some of the advantages of buying a franchise? What might be some drawbacks?

- In the next video clip you’re going to be looking at a very small franchise from two points of view—the business owners and the people that buy franchises from them. Watch for ways that both groups try to make money (profit) from a franchise.

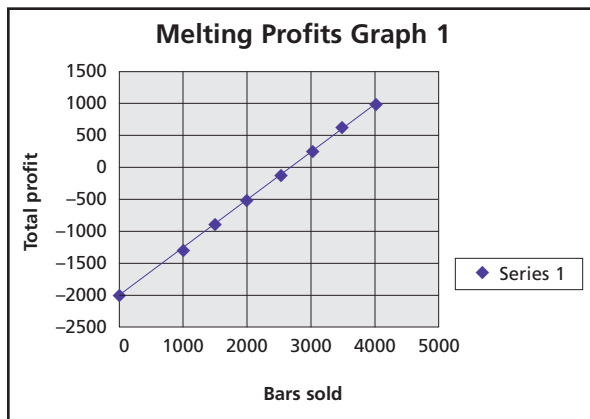
CUE the video to approximate time code 12:50 (black screen before the segment titled “Melting Profits”) and PLAY to approximate time code 16:02. PAUSE the video when you see the on-screen graphic of the formula for profits (see below), and after the host says “. . . that’s the number of bars sold, minus 2,000.”

$P = .75N - 2000$ <p>P = Profit N = Number of bars sold</p>
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Point out that, although the video formula uses  $N$  as the unknown number instead of  $x$ , the function works in the same way. Ask students how they might rewrite the formula to better fit the linear function formulas they’ve seen so far ( $y = .75x - 2000$ ). Remind students that in the linear function,  $f(x) = mx + b$ , “.75” is fixed amount “ $m$ ” (the amount of profit the fruit-bar stand owners keep for each bar sold) and 2000 is fixed amount “ $b$ ” (the up-front franchise fee). In place of the  $N$  used in the video, we used  $x$ .

Ask for volunteers to come to the board and work on the solution set for different values of  $x$ , beginning with zero bars sold, jumping to 1,000 and working at least five problems, ending with 4,000 bars sold. As each equation is solved, begin creating a solution set of ordered pairs for this problem. (NOTE: Depending on the skills and abilities of your students, you may wish to have them multiply the decimal calculations manually or with a calculator. A projection calculator will help all students see the work, or you might hand out calculators to everyone so that the whole class can practice the equations.)

Create a coordinate graph and plot the ordered pairs, using a straight edge to connect the points (or project the sample graph shown here).



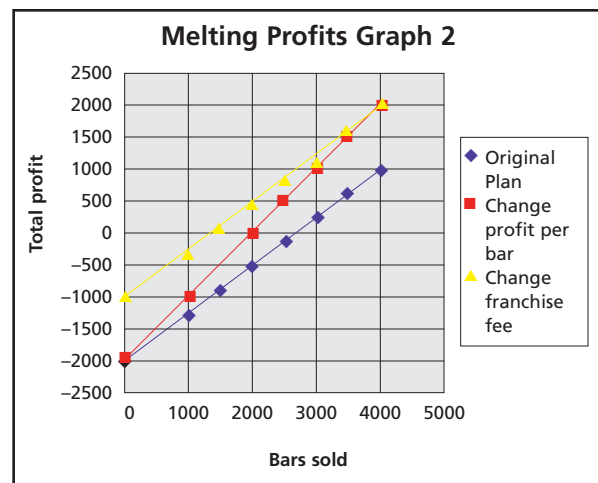
Lead a brief discussion about the information shown on the graph.

- Is this a linear equation? How do you know?
- What does it mean when a point falls below the  $x$ -axis? What does a negative number mean to a businessperson in this case?
- Why is the franchise owner at negative \$2,000 when he first opens his fruit-bar stand?
- Estimate at about which point on the  $x$ -axis the profits “break even,” or reach a  $y$ -value of zero (between 2666 and 2667).
- Why do you think people weren’t interested in buying into this franchise? How long do you think it would take each fruit-bar stand owner to sell that many bars?

The advisor gave the business owners two options to improve their business: Raise the franchise owners’ profit per bar—the fixed amount we call “ $m$ ” (\$0.75); or lower the franchise fee—the “ $b$ ” value (\$2,000). Before continuing with the video, divide the class into two groups: the first will change the  $m$ -value to \$1.00, create a new solution set, and graph the linear equation. The other group will change the

$b$ -value to \$1000. (NOTE: You may wish to have students work in pairs or small groups on this activity.) If you haven’t already done so, pass out graph paper and straight edges (and calculators if you wish). Walk around the room checking students’ work to make sure they are calculating correctly.

Have groups share their results and graphs with the class and lead a discussion about which option would work best, both for the girls who own the business and for their franchise owners. You may wish to project some student graphs for comparison, or use the following example that shows both options along with the original plan.



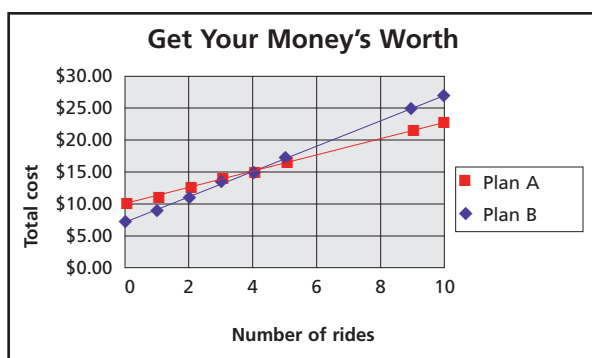
Return to the video and PLAY until the end of the program, STOPPING at approximate time code 18:30 (black screen). Discuss the solutions tried in the video. Point out the steeper slope on the line for the change in profit per bar, which proves the consultant’s point about the sharp increase in profits over time.

### Homework

Pass out copies of the **Get Your Money’s Worth** worksheet and read the problem aloud. Explain that this problem is similar to the video problem they just solved together—in the video the franchise owner had a fee plus a cost per bar. In this worksheet, students have an admission price

plus cost per rides. Assign this worksheet as independent work to do in class or at home.

Check the papers together in the next class session. Students should notice that Plan B's line with the steeper slope costs less for fewer rides, but once the lines cross at the coordinate (4, \$15.00), Plan A quickly becomes the better deal. Point out that this graph shows the better deal as the less-steep slope, which is different from the Melting Profits graph. Ask for volunteers to explain why you would want a steep slope for the franchise profits but not for this amusement park cost.



ANSWER KEY:

- Solution sets might include the ordered pairs.  $\{(1, 9.00), (2, 11.00), (3, 13.00) \dots (9, 25.00)\}$
- Solution sets might include the ordered pairs.  $\{(1, 11.25), (2, 12.50), (3, 13.75) \dots (12, 25.00)\}$
- For riding four rides, the plans cost the same: \$15.00.
- Plan A is the better plan for a \$25.00 limit. It allows 12 rides rather than nine.

## Assessment

### Individual Assessment

Check students' understanding of the concepts of function and graphing linear equations from

the two assignments **The Input/Output Machine** and **Get Your Money's Worth**. In addition, ask students to write a reflection about what they learned from examining the graphs in the second worksheet. They should consider how one plan may be better than another in different situations and why data must be analyzed to understand the problem fully.

### Group Assessment

Assess the group work completed during the *Mathemedia* video problem from Day Three. Look for evidence of collaborative problem solving as well as the students' understanding of linear functions and equations, ordered pairs, solution sets, and coordinate graphing.

## Extension Activity: Mystery Linear Equations in Spreadsheet Software

Divide the class into pairs or groups of three and have each group use the graph and chart features of spreadsheet software (e.g., Microsoft Excel, Appleworks, Apple iWork Numbers, ClarisWorks) to create mystery linear equations for other groups to solve.

Use a computer with presentation software or a computer lab to lead students through the process of creating x-y graphs with spreadsheet software. (NOTE: The following instructions were written for Microsoft Excel 2003, but can be easily adapted for other editions, software, or platforms.)

- Open a blank worksheet. Explain to students that you're going to add ordered pairs in the first five rows of the worksheet. Column A will represent the  $x$ -values and column B will hold the  $y$ -values.
- Enter the following numbers in the two columns.

	Column A (x-value)	Column B (y-value)
1	2	6
2	3	10
3	4	14
4	5	18
5	6	22

3. Select both sets of data. (For example, by clicking on the “2” and dragging the mouse to the “22,” or however you prefer to select a block of cells.)
4. Click on Insert/Chart. A wizard window will appear that walks users through a series of four steps to create a variety of charts. Use the following instructions to create a linear graph.

Step 1: Select “X/Y (Scatter)” from the choices in Chart Type, and in the Chart Sub-type area select the type that provides “Scatter with data points connected by lines.” Click NEXT.

Step 2: Make sure the “Series in” is set to column, and select NEXT again. (Note the “series” tab at the top of this step that will allow users to add additional lines for other equations, but ignore that step for now.)

Step 3: This step allows users to add a title, select different types of grid lines, and perform other tasks to add to the visual appeal of the graph (point out the tabs across the top). Type the words “What’s my function?” in the

$$\begin{array}{r}
 2a + 4b = 18 \\
 2a - 3b = 4 \\
 \hline
 7b = 14 \\
 b = 2
 \end{array}$$

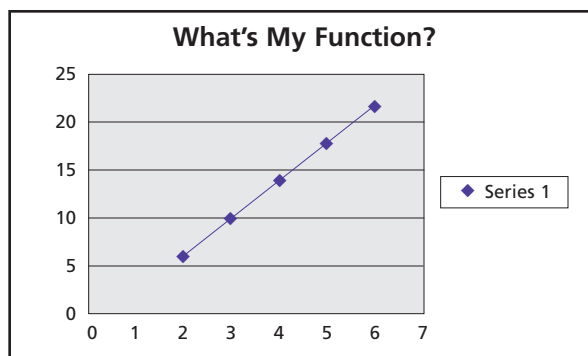
“Mathematicians do not study objects, but relations between objects.”

—Henri Poincare, mathematician

Chart Title field, ignore all of the other options, and click NEXT.

Step 4: In the final step users can choose where to place the graph—in a new sheet or as an object in the current sheet (default). Leave it as the default (object in) and click FINISH.

5. The result is an x/y grid with the ordered pairs plotted on the graph, as in the illustration below. Students can copy and paste the graph into a word processing program or other presentation software. Show them how to click on the graph to select it and then copy/paste using the edit menu or keyboard shortcuts.



6. Ask for volunteers to identify the solution set as well as the function represented by this graph. The solution set for this graph is:

$$\{(2, 6), (3, 10), (4, 14), (5, 18), (6, 22)\}.$$

Using trial and error, students should be able to identify this simple linear function as

$$f(x) = 4x - 2.$$

(NOTE: This activity is designed for beginning algebra students to use the trial-and-error method for identifying functions. It can also be used for more advanced students practicing slope-intercept calculations not covered in this lesson.)

Explain that you want each set of partners to create a linear equation and use the spreadsheet program to plot at least five ordered pairs into a coordinate grid similar to this one. Allow time for interested students to try out various software features to improve the look/feel of their finished coordinate graphs. Then have them copy and paste the graph into a word document or drawing software and create a Mystery Equation poster. Post the graphs and allow time for other students to try to identify the linear function or rule that delivers each plotted graph.

## Resources

Students who require additional practice with the concepts or skills covered in this lesson might benefit from one or more of these online interactives and games.

[http://nlvm.usu.edu/en/nav/frames\\_asid\\_191\\_g\\_3\\_t\\_1.html](http://nlvm.usu.edu/en/nav/frames_asid_191_g_3_t_1.html)

**Function Machine.** Have students look for patterns and functions with this interactive function machine from the National Library of Virtual Manipulatives (NLVM), hosted by Utah State University.

[http://nlvm.usu.edu/en/nav/frames\\_asid\\_109\\_g\\_4\\_t\\_2.html?open=activities&from=category\\_g\\_4\\_t\\_2.html](http://nlvm.usu.edu/en/nav/frames_asid_109_g_4_t_2.html?open=activities&from=category_g_4_t_2.html)

**Grapher.** At another interactive site from the NLVM, students can manipulate variables to investigate functions on a coordinate plane.

<http://www.shodor.org/interactivate/activities/SimpleCoordinates/>

**Simple Coordinates Game.** This interactive from Shodor Interactivate allows students to practice identifying the coordinates of items located in the first quadrant on a coordinate plane (avoiding negative values for x or y).

<http://www.quia.com/rr/4096.html>

**From Rags to Riches Game.** Play this interactive game in which students solve one- and two-step linear equations and simple algebraic proportions as they work toward winning \$1,000,000. (NOTE: Some of the equations involve math properties not covered in this lesson, including the Distributive and Associative Properties.)

<http://funbasedlearning.com/>

**Assorted Algebra Games from**

**FunBasedLearning.** Select the game and level that best fit students' abilities and needs.

- Practice plotting points on a coordinate plane with three different levels for the **Graph Mole**. Whack the mole by selecting the ordered pair that identifies his location on the coordinate plane. (Easy, Medium, or Hard levels)
- Learn to draw linear equations by finding points with the **Line Gem 1** game.

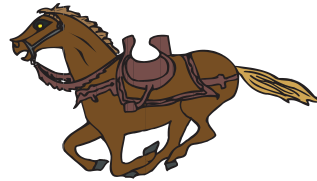
# For Want of a Nail

## A Nursery Rhyme

For want of a nail the shoe was lost.



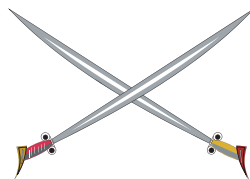
For want of a shoe the horse was lost.



For want of a horse the rider was lost.



For want of a rider the battle was lost.



For want of a battle the kingdom was lost.



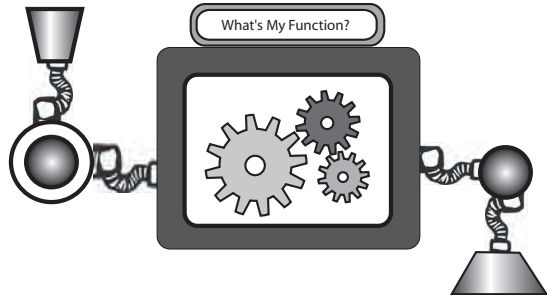
And all for the want of a horseshoe nail.

# The Input/Output Machine

Name \_\_\_\_\_

Date \_\_\_\_\_

A. Robin Hood fooled King John when he gave him  $\frac{3}{6}$  of the coins. You could say that he input the fraction  $\frac{1}{2}$  ( $x$ ) and his output was  $\frac{3}{6}$  ( $y$ ). The function that caused that output was multiplying both the numerator and the denominator by 3.



$$\frac{1}{2} \cdot \frac{3}{3} = \frac{3}{6}$$

1. Write the function for the machine that produces these ordered pairs. \_\_\_\_\_

2. List the outputs, or  $y$  values, that would complete the following solution set, using this machine's function.

$$\{(2/3, \underline{\quad}), (1/6, \underline{\quad}), (3/7, \underline{\quad})\}$$

B. Figure out the function that was used to produce each of the following solution sets. Complete ordered pairs in each solution set and write its function using mathematical symbols.

1.  $\{(2, 9), (10, 17), (8, 15), (3, \underline{\quad}), (21, \underline{\quad})\}$  Function: \_\_\_\_\_

2.  $\{(21, 13), (10, 2), (64, 56), (19, \underline{\quad}), (12, \underline{\quad})\}$  Function: \_\_\_\_\_

3.  $\{(7, 21), (12, 36), (2, 6), (11, \underline{\quad}), (5, \underline{\quad})\}$  Function: \_\_\_\_\_

4.  $\{(10, 2.50), (5, 1.25), (16, 4.00), (8, \underline{\quad}), (100, \underline{\quad})\}$  Function: \_\_\_\_\_

5.  $\{(\frac{1}{2}, \frac{2}{6}), (\frac{2}{3}, \frac{4}{9}), (\frac{1}{3}, \underline{\quad}), (\frac{2}{5}, \underline{\quad})\}$  Function: \_\_\_\_\_

6.  $\{(\frac{2}{3}, \frac{11}{12}), (\frac{1}{5}, \frac{9}{20}), (\frac{1}{2}, \frac{3}{4}), (\frac{1}{16}, \underline{\quad})\}$  Function: \_\_\_\_\_

7.  $\{(3, 7), (10, 21), (4, 9), (5, \underline{\quad}), (7, \underline{\quad})\}$  Function: \_\_\_\_\_

8.  $\{(2, 19), (5, 49), (3, 29), (7, \underline{\quad}), (10, \underline{\quad})\}$  Function: \_\_\_\_\_

**Bonus Question:** Is it possible to have the same output ( $y$ ) for two different inputs ( $x$ ) in any solution set? For example:  $\{(2, 9), (6, 9)\}$ . How do you know? Write your answer and explanation on the back of this paper.

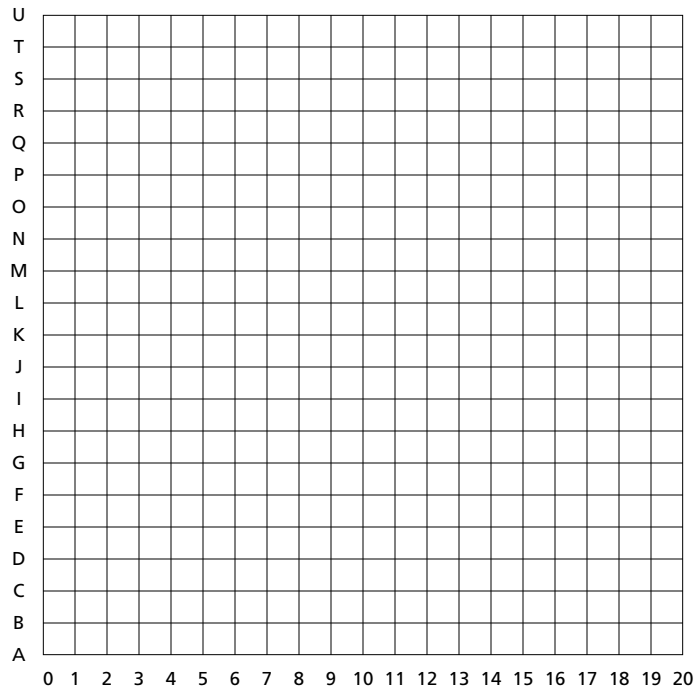
# André's Trip Home

Name \_\_\_\_\_

Date \_\_\_\_\_

Sometimes getting home isn't easy. André just returned from a trip to visit his grandmother and aunt. He arrived at the railroad station in his hometown. Then his trouble began. He had planned to take the subway, but it was closed because of a strike. He waited for a bus, but it never came. Finally he decided to take a taxi. The taxi immediately got stuck in heavy traffic. In an effort to escape the gridlock, the driver turned into another street, which was blocked because of a fire. Heading in a different direction, the taxi got stuck again because a water main had burst, backing up traffic. The cab finally got through the flooded area, but then, only a few blocks farther, there was a detour because of road work. Next the taxi came to another street that was blocked off because of a collision. André felt lucky to get home safely.

Now you plot André's trip home in the taxi by putting a dot on the graph at each of the following points and then by joining your dots with a line.



(From *Mathemedia*,  
"Coordinates" Student  
workbook, page 12-2.)

- |                           |         |          |                 |
|---------------------------|---------|----------|-----------------|
| 1. 0,A (Railroad station) | 6. 6,F  | 11. 11,K | 16. 15,P        |
| 2. 3,B                    | 7. 7,G  | 12. 13,L | 17. 15,Q        |
| 3. 2,C                    | 8. 8,H  | 13. 15,M | 18. 18,R        |
| 4. 5,D                    | 9. 10,I | 14. 13,N | 19. 19,S        |
| 5. 4,E                    | 10. 9,J | 15. 12,O | 20. 20,T (Home) |

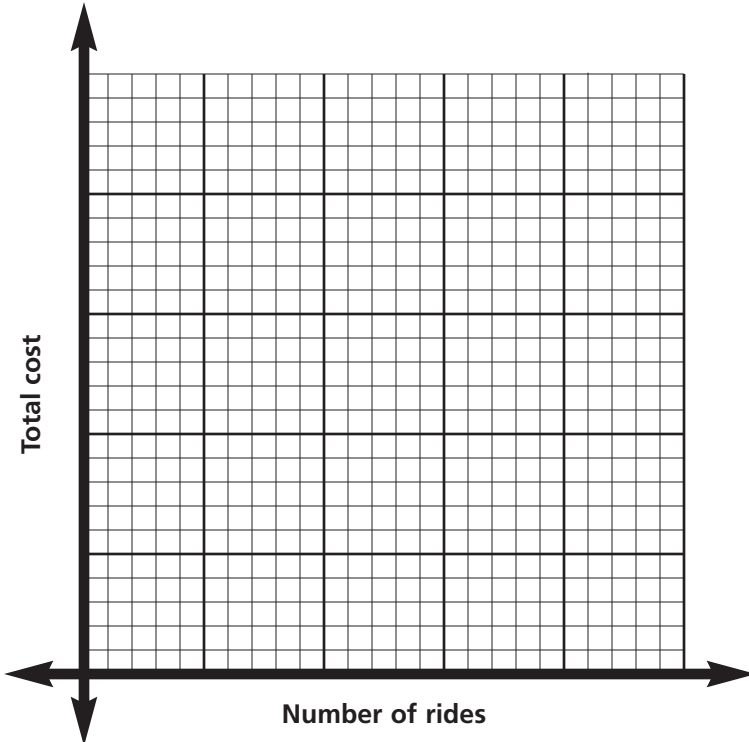
Suppose each of the lines marked with a number or a letter of the alphabet were really a city street. While the line you have drawn cuts through the middle of city blocks in a way that a taxi never could, all of your dots show intersections where streets cross. A taxi could take a route that goes through all the intersections you have marked.

# Get Your Money's Worth

Name \_\_\_\_\_

Date \_\_\_\_\_

A new amusement park has just opened in your town and you want to make sure you get as many rides as possible for your money. The park has two cost plans for visitors. Each plan includes a fee for admission and an additional charge for each ride. It's up to you to decide which plan works best for you. Check out Plan A and Plan B in the boxes beside the graph.



PLAN A	
Price of admission:	\$10.00
Cost per ride:	\$ 1.25

PLAN B	
Price of admission:	\$7.00
Cost per ride:	\$2.00

Use the linear equation formula to create a solution set containing at least five different solutions—changing the number of rides you can go on in each plan. Add numbers to the x and y axis of the coordinate plane above and graph the solution set for Plan A with a red line and Plan B with a blue or black line. Then examine the lines to see which plan works best for you.

$y = mx + b$ <p>m = cost per ride b = admission price</p>
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1. Solution set for Plan A \_\_\_\_\_

2. Solution set for Plan B \_\_\_\_\_

3. At what number of rides are the two plans exactly the same cost? \_\_\_\_\_